

Problem. (*American Mathematical Monthly*, March 2004. Problem 11068, proposed by Herbert Wilf, University of Pennsylvania.) For a rational number x that equals a/b in lowest terms, let $f(x) = ab$.

(a) Show that

$$\sum_{x \in \mathbb{Q}^+} \frac{1}{f(x)^2} = \frac{5}{2},$$

where the sum extends over all positive rationals.

(b) More generally, exhibit an infinite sequence of distinct rational exponents s such that $\sum_{x \in \mathbb{Q}^+} f(x)^{-s}$ is rational.