Introduction

Let K be an algebraic number field over Q and O_K its ring of integers. It is well known that O_K possesses a free Z-basis. If this integral basis is generated by a single element, i.e. $O_K = Z[\alpha]$ for some element $\alpha \in O_K$, then O_K is said to possess a "power basis." The question of the existence of a power basis was originally examined by Dedekind, who found a now classic (non-abelian) cubic field which has no power basis. The Dedekind test involves the splitting of primes $p < n$, the degree of the extension, which for cubic fields reduces to consideration of the prime 2. In this paper we construct infinitely many (abelian) cubic fields K which do not satisfy the criterion necessary for the classical Dedekind test, and yet still do not possess a power basis. The fields are constructed as cubic subfields of prime cyclotomic fields. We also demonstrate a connection between indices and norms of "elements" (integers with trace zero).

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