Introduction

This paper is concerned with five Banach space properties - the Dunford-Pettis property, the Reciprocal Dunford-Pettis property, the Dieudonné property, Pelczynski's property V, and Grothendieck's property. These five properties are related in that they each give either a necessary or sufficient criterion for weak compactness in the dual of the space. We first study the properties in the setting of an arbitrary Banach space in Part I. Section 1 defines the properties and gives various equivalent forms. We also note two easy implications - both property V and the Dieudonné property are stronger than the Reciprocal Dunford-Pettis property. In Section 2 we present a new way of looking at property V. This shows its similarity to the Dieudonné property and in fact shows V is a stronger property. Section 3 presents more equivalent forms of these two properties. It ends with a proof that a Banach space which is a dual space and has property V is a Grothendieck space. In Section 4 we study some of the consequences of a space enjoying the properties. Part I ends with a collection of examples in Section 5.

Part II is concerned with a certain class of examples - C^* and W^* algebras. We are able to settle all of the questions for a particular C^* algebra, the compact operators on a Hilbert space, in Section 1. This C^* algebra enjoys property V, the Dieudonné property and the Reciprocal Dunford-Pettis property. It fails to enjoy the Dunford-Pettis property and Grothendieck's property when the Hilbert space is infinite dimensional. In Section 2 we turn to arbitrary C^* algebras. We show they enjoy the Reciprocal Dunford-Pettis property and present possible tools for working on the other questions. Finally Section 3 presents some techniques for constructing potential counterexamples.

X, Y, and Z will always denote Banach Spaces. An expression like $T : X \to Y$ will implicitly mean T is a bounded linear operator from X to Y. Topological duals will be denoted by *'s; for example, X^* is the dual of X. The weak topology will always refer to the topology given by the dual of the space. If the space has a predual the topology it induces will be referred to as the weak * topology.

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