

Results are derived on solutions to  $AA^\dagger = mI_h$  in the  $n$ th cyclotomic field, where  $A$  is square. When  $m=h$  these give a nonexistence theorem for certain generalized Hadamard matrices. Earlier results by Marshall Hall on Hadamard completions are generalized. In the process theorems of Witt and of Hall and H. J. Ryser are also generalized.

## 1. Introduction

A  $(1, -1)$  matrix  $H$  of order  $h$  is called a Hadamard matrix if  $HH^T = hI_h$  [20]. Butson [13] defines a generalized Hadamard matrix (GHM) as a matrix  $H$  of  $n$ th roots of unity for which  $HH^\dagger = hI_h$ . A Hadamard matrix of order  $h$  has  $h=1, 2,$  or  $h \equiv 0 \pmod{4}$ . Similarly, if  $n$  is prime, a GHM of order  $h$  has  $h=1$  or  $h \equiv 0 \pmod{n}$ . The basic conjecture on Hadamard matrices is that they exist for all orders  $h \equiv 0 \pmod{4}$ . When  $n$  is an odd prime, the corresponding conjecture for GHMs is that they exist for all orders  $h \equiv 0 \pmod{n}$  for which  $I_h \stackrel{c}{=} hI_h$ . We shall exhibit a new GHM of order 12 of cube roots of

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