

I originally started this investigation after working my way through a book on complex analysis. As the extension of differentiation from the real numbers to the complex numbers leads to such a rich theory, I wondered what would happen if it were further extended to the quaternions, which I knew were somehow an extension of the complex numbers. After defining differentiation and finding useful criteria for differentiability, I started testing functions for differentiability. Unfortunately most functions turned out not to be differentiable, except at a small set of points. This was a source of confusion and puzzlement, until I was able to show that the only functions that were differentiable on an open, connected set were of the form $z \mapsto \alpha z + \beta$. This started me wondering why a function as basic as $z \mapsto z^2$ wasn't differentiable on an open set. Not surprisingly, this led me to investigate the algebraic properties of the quaternions. I was able to prove several interesting things about the quaternion skew field, including finding all its automorphisms, finding all the normal subgroups of its "unit sphere", and finally understanding why $z \mapsto z^2$ wasn't differentiable on an open set. This paper will show, in the first part, why the differentiable quaternion functions are essentially trivial, and in the second part will detail the algebraic results I got.