Abstract

The planar circular restricted three-body problem (PCR3BP) is a standard example of a system with two degrees of freedom that is not integrable. Despite the efforts of many great mathematicians for over the past two centuries, our understanding of the solutions of PCR3BP is still far from complete. One way to understand the behavior of such solutions is through the investigation of invariant sets, which in this case, consists of periodic and quasiperiodic solutions. Up to now, such investigations have relied upon the highly successful KAM theory, developed only about fourty years ago. However, such applications of KAM theory are limited since KAM theory only describes systems that are close to integrable. This places strong constraints on the Jacobi constant and mass ratio of PCR3BP. Furthermore, KAM theory can prove only the existence of quasiperiodic solutions.

In this paper, we prove the existence of both quasiperiodic and periodic solutions for PCR3BP. Moreover, our results hold for a much larger, more practical range of Jacobi constant and mass ratio than that required for KAM theory. In fact, we show that periodic and quasiperiodic solutions exist for parameters of PCR3BP that approximate a "Sun–Jupiter–planar Pluto" system. Our approach, however, is based upon the more recent and less well-known Aubry-Mather theory. Aubry-Mather theory is a mathematical theory that provides a framework for constructing periodic and quasiperiodic solutions for Hamiltonian systems with two degrees of freedom. Moreover, these systems need not be near-integrable. By constructing an appropriate Poincaré return map on a surface of constant energy, we are able to examine the dynamics of PCR3BP on a two-dimensional section, thereby giving us an appropriate setting for the application of Aubry-Mather theory and enabling us to prove the existence of a rich variety of periodic and quasiperiodic motions.