Introduction

The congruent number problem is a classical Diophantine problem to determine all positive integers that are the area of some right triangle with rational side lengths. It is well known that a positive integer n is congruent if and only if the elliptic curve $E^n : y^2 = x^3 - n^2x$ has positive Mordell-Weil rank, which, assuming the Birch and Swinnerton-Dyer conjecture, is equivalent to the twist of the L-function of E^1 by the quadratic character χ_n has vanishing central value. Tunnel, in his classical paper (Tunnell, 1983), proved an effective criterion for congruent numbers by finding the preimages of the modular form associated to E^1 under the so-called Shimura correspondence between modular forms of half integral weights and integral weights, which combined with Walspurger's theorem, gives a easy criterion for the vanishing of the twisted central L-values.

It would be nice to generalize the congruent number problem to real quadratic fields, and one of the simplest case is to $F = \mathbb{Q}(\sqrt{5})$ which has small discriminant (5) and narrow class number 1. We say an element α in the ring of integers of F is F-congruent if it is the area of some right triangle with side lengths in F. Work has already been done on determining whether a rational integer is F-rational (c.f. (Tada, 2001)). Furthermore, we still have the analogous result that α is F-congruent if and only if the group $E^\alpha(F)$ of F- rational points has positive rank ((Tada, 2001) Theorem 1; the proof there is for rational integers α but goes through without change for F-integers; note that this result fails if we change F to $\mathbb{Q}(\sqrt{2})$, in which case $E^\alpha(F)$ has torsion points that are not 2-torsion). Therefore we could still attack the problem in the same way Tunnel did: that is, try to find Shimura lifts of the Hilbert modular form associated to the elliptic curve E^1 base-changed to F. In this paper we explicitly construct such a weight 3/2 Shimura lift using theta series of ternary quadratic forms. The first three sections are expository,

which deals with the necessary background and collects related results without proof. These results are well-known and we give complete reference in case the readers want the details. The first section contains the definitions and basic properties of integral weight and half integral weight Hilbert modular forms, as well as the Shimura correspondence. The second section deals with about quaternionic modular forms and Brandt matrices. In the third section we present the result of (Sirolli, 2014) on ternary theta series, which is our main tool of constructing Shimura lifts. Finally, in the last section we apply the previous results to compute the Shimura lift with the aid of MAGMA(Bosma et al., 1997), and then derive certian partial results on deciding F-congruent numbers.