

Extensions of Arithmetic Geometric Mean (AGM)

Problem:

Can the Arithmetical-Geometric Mean of two numbers be extended to n numbers?

Introduction:

One of the classical problems in the realm of analysis is that of finding a closed form expression for the Arithmetic-Geometric Mean of two numbers, $M(a,b)$. An author was introduced to this problem in a short course on introductory elliptical integral theory at a summer program. The teacher of the course, Dr. George Shabat of the Institute for Mathematical Studies, Moscow, proposed an extension to this problem and also gave other background information and suggestions.

In this paper, the authors propose another extension to this problem and partially treat it and Dr. Shabat's proposal. The primary excitement about this problem is that computer tools, such as *Mathematica*, can be used to perform some of the difficult calculations that obviously hindered progress on this problem in earlier times. Though significant proofs or results are not independently produced here, the ultimate goal of this paper is to show how a calculating tool can help in the exploration of a very beautiful problem.

This paper is divided into three parts. The first part is a discussion of the Arithmetical-Geometric Mean of two numbers. Most of this treatment can be found in the references cited below, though some of the basic results presented here are our own derivations. The second part is a discussion of the Arithmetical-Geometric Mean of three numbers, Dr. Shabat's proposal for an extension of the Arithmetical-Geometric Mean. And the final section is an attempt to generalize the Arithmetical-Geometric Mean, with numerical evidence, to arbitrary numbers of values using different orders of the symmetric mean.