

Problem: Let g be the sequence given by $g(0) = 0$ and $g(n+1) = (n+1)^2g(n) + (n!)^2$ for $n \geq 0$. Show that if p is a prime larger than 3, then $p \mid g(\frac{p-1}{2})$.¹

Problem: Let k and n be positive integers. Let $I(k, n) = \{j \in \mathbb{N} : k^n < j < (k+1)^n\}$.

(a) For $n = 2$ and all k , prove that there do not exist distinct $a, b \in I(k, n)$ such that ab is a square.

(b) For each $n > 2$, prove that when k is sufficiently large, there exist n distinct integers in $I(k, n)$ whose product is the n th power of an integer.¹