Regularity for the Monge-Ampère equation, with applications to the semigeostrophic equations

The Monge-Ampère equation arises in connections with several problems from geometry and analysis (regularity for optimal transport maps, the Minkowski problem, the affine sphere problem, etc.), and its regularity theory has been widely studied. However, a natural question that remained open for long in the theory was the Sobolev regularity of solutions when the right hand side is merely bounded away from zero and infinity. Apart from its own interest, this question naturally arises in the theory of existence of global solutions to the semigeostrophic equations. The latter are a simple model used in meteorology to describe large scale atmospheric flows. In these lectures we will introduce the semigeostrophic equations, discuss their relation with Monge-Ampère, show how to obtain Sobolev regularity estimates, and finally prove global existence of solutions to the semigeostrophic equations.